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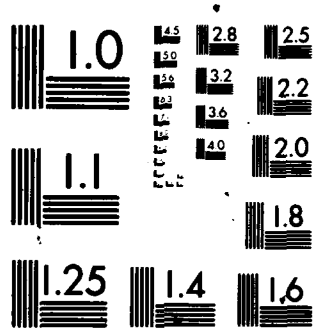

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A new parameter adaptive control shceme for linear multivariable systems has been developed. It is felt that such an adaptive controller could be used in a variety of aerospace applications. A "multi-purpose" controller has been designed which simultaneously decouples, places poles arbitrairily, rejects disturbances, insures zero error tracking, and is robust with respect to parameter variations. A new and straightforward method for obtaining simple low order models of systems whose dynamical behavior approximates that → next page

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of more complex, higher order systems has also been developed. Such low order models can be used in the design of low compensators for the more complex systems. A complete new resolution has been presented to the question of what changes occur to the individual transfer matrix elements of a linear multivariable system under local, scalar output feedback. In particular, it has been shown what poles become controllable and observable via any input/output pair when constant gain output feedback is applied between any (i-th) output and any (j-th) input. THe question of parameter variation and the development of compensators which are insensitive to that variation has been resolved for a specific feedback group and is being studied for systems with parameters. Results have also been obtained for the pole-assignment problem involving parameters using intersection theory and some preliminary work has been done on the realization, coprime factorization and trace assignment problems for systems defined over the polynominal ring in n-variable over the integers.

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PRACTICAL METHODS FOR THE COMPENSATON

CONTROL OF MULTIVARIABLE SYSTEMS

AFOSR-77-3182

RESEARCH OBJECTIVES

The primary objective of this research is the development of practical methods for the compensation and control of multivariable systems. Success in this area would facilitate the design of controllers for complex Air Force systems. A variety of different approaches have been employed to accomplish this objective.

In particular, we were most successful in developing a new parameter adaptive control scheme for linear multivariable systems. The design algorithm employs an adaptive Luenberger observer which automatically adjusts the poles of a given system. It is felt that such an adaptive controller could be used in a variety of aerospace applications.

We were also able to show that "multi-purpose" controllers can be designed which simultaneously perform a variety of control functions. In particular, we constructively demonstrated how to build a controller which simultaneously decouples, places poles arbitrarily, rejects disturbances, insures zero error tracking, and is robust with respect to parameter variations. We also presented a new and straightforward method for obtaining simple low order models of systems whose dynamical behavior approximates that of more complex, higher order systems. Such low order models can be used in the design of low order compensators for the more complex systems.

A complete new resolution was also presented to the question of what changes occur to the individual transfer matrix elements of a linear multivariable system under local, scalar output feedback. In

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particular, it was shown what poles become controllable and observable via any input/output pair when constant gain output feedback is applied between any (i-th) output and any (j-th) input. The changes which simultaneously occur to the numerator elements of the transfer matrix were then determined through the employement of some new relationships derived from an appropriate relatively right prime factorization of the system transfer matrix.

Another research objective is the study of parameterized system models with a view towards developing compensation and control techniques which are particularly crucial; namely, (i) the question of parameter variation and the development of compensators which are insensitive to that variation; and (ii) the question of system structure and qualitative properties for parameterized models. The techniques considered involve the methods of algebraic geometry and revolve around three key questions: (i) can the orbits in the space of linear systems under equivalence via the action of an algebraic group be described and classified?, (ii) what spectral structures can be achieved through the use of compensation?, and, (iii) what are the essential elements required in extending results to domains other than the real and complex numbers? Question 1 has been resolved for the feedback group and is being studied for systems with parameters. Results have been obtained for the pole-assignment problem involving parameters using intersection theory and some preliminary work has been done on the realization, coprime factorization and trace assignment problems for systems defined over the polynomial ring in n-variables over the integers.

STATUS

Adaptive Control

Although considerable progress has been made in designing globally stable adaptive control schemes for unknown continuous time single input-single output (scalar) systems, little progress has been made toward the extension of these ideas to the multiple input-multiple output (multivariable) case. In [1] an equation is derived that can be used to identify a set of controller parameters which completely assign all poles and zeros of a linear multivariable systems represented by a (pxm) strictly proper transfer matrix T(s) when $p \geq m$ and only input-output data is available for measurement. This equation is used to derive a parameter adaptive control system for linear multivariable systems.

The control structure contains an adaptive Luenberger observer which assigns as poles of the closed loop system the zeros of the unknown system (plant), and possibly some additional poles specified by the designer. The new overall system zeros and the remaining poles are obtained by use of a fixed precompensator which can be arbitrarily specified by the designer. Since the incorporated parameter identifier directly estimates the control parameters without explicitly identifying a parameterized model of the unknown plant the structure can be classified as a "direct adaptive controller."

Since the plant zeros are assigned as closed loop poles, to construct the controller one must be assured that the plant's open loop zeros lie strictly in the left half plane. In addition, to complete the design one must know, *a priori*, an upperbound on the plant's observability index, as well as the structure of the "interactor matrix" associated with T(s). In many cases, this latter requirement is essentially equivalent to knowledge of the smallest relative degree in each row of T(s). To assure proper

performance, additional information about the plant transfer matrix structure is needed. This information is analogus to information on the high frequency gain necessary for adaptive control of scalar systems. It might be noted that the design presented in [1] is generally applicable to a large class of multivariable systems.

Multi-Purpose Controllers

Frequency domain methods have always dominated control system design in the scalar (single input/output) case, when compared to the more "modern" state-space or differential operator methods, due to the relative simplicity of the resulting controllers and their ability to function acceptably over a rather wide range of plant parameter variations; i.e. their robustness. It is not surprising, therefore, that numerous studies have been made to "extend" various frequency domain techniques to the multivariable case in order to simply and reliably achieve a diversity of desired design goals. In most cases, however, direct extensions of scalar frequency domain procedures, such as the Nyquist stability criteria or the root locus, are not possible and often rather complex modifications have to be made to existing theories in order to achieve appropriate design objectives. Further complicating the picture is the fact that noninteraction (or decoupling) is often an additional design objective in the multivariable case, and a completely decoupled, stable system cannot always be achieved by the relatively simple feedforward controllers obtained by multivariable, frequency domain methods.

On the other hand, the so called "modern" methods which have generally relied on exact knowledge of the plant, are continually being improved upon and extended to take into consideration parameter uncertainty and/or variations;

i.e. robustness is becoming increasingly important in designs based on state-space or differential operator methods. Although these "modern" methods generally imply more complex controller configurations, than those associated with frequency domain methods, they are less heuristic to implement and can generally achieve more than is possible with the simpler controllers designed by frequency domain methods. Moreover, with the ever increasing utilization of computers in the control loop, it may be argued that controller simplicity is no longer as important as it once was, and one might therefore expect to see more complex controllers being used in future applications.

In the light of these observations, a new procedure has been developed for designing controllers which simultaneously achieve a variety of desired design goals in deterministic, unity feedback, linear multivariable system. More specifically, in [2] a new algorithm is presented for the systematic design of a "three part" multivariable controller which simultaneously insures

(a) a noninteractive or decoupled closed loop design,

(b) complete and arbitrary closed loop pole placement, which implies desired (single loop) transient performance as well as closed loop stability,

(c) zero steady-state errors between the plant outputs and any nondecreasing deterministic inputs,

(d) complete steady-state output rejection of nondecreasing deterministic disturbances, and

(e) robustness with respect to stability, disturbance rejection, and zero error tracking for rather substantial plant parameter variations.

Our development in [2] employs the more "modern" (Laplace transformed) differential operator approach for controller synthesis, which involves transfer matrix factorizations and the manipulation of polynomial matrices in the Laplace operator s.

Low Order Models

The problem of finding reduced order models for high order systems, sometimes referred to as the "model reduction problem", is an important one to the practicing engineer since it is difficult to apply the design procedures of modern and classical control theory to high order systems. Numerous solutions to the model reduction problem have been proposed during the past two decades. Many of these are based on first deriving transfer function or state-space models for the high order systems and then simplifying these models. Other methods use time or frequency response data to directly fit low order models. Our procedure falls into this latter category, since it determines a model which matches the frequency response of the original high order system at a certain set of prespecified frequencies. Its primary advantage lies in the simplicity of implementation. In particular, no intermediary high order model need be calculated, only one test input need be used, and the calculation of model parameters only requires the solution of a simple set of linear equations. The model parameters can also be obtained as the output of an analog adaptive network, since the algorithm makes use of the generalized equation error identification scheme due to Lion. Most importantly, our algorithm readily generalizes to the multiple input-output (multivariable) case where classical frequency and time domain procedures become cumbersome to apply.

More specifically, in [3], we present a new method for obtaining simple low order models whose dynamical behavior approximates that of more complex, higher order, stable linear systems. The low order model is determined by applying an identification procedure to input-output data obtained by "driving" the original system with a special periodic test signal. We prove that in the scalar case a Lion-type model adjustment identifier will determine a constant k^{th} order model of an n^{th} order $(k < n)$ system provided the system input consists of exactly k distinct sinusoids. This K^{th} order model will approximate the higher order system in the sense that its frequency response matches that of the model at the k input frequencies (provided the model obtained is stable). We then show that this result can be extended in a very natural way to the multivariable case. We finally demonstrate by example that this procedure can produce excellent low order models when such models exist.

<u>Decentralized Control</u>

In the control of linear multivariable systems, constraints are often imposed on both the complexity of the controller as well as its placement relative to the system's inputs and outputs. An interesting question relative to this observation is what changes occur to the dynamical relations between any input/output pair as the result of applying constant gain output feedback between any (i-th) output, y_i and any (j-th) input, u_j This question is clearly related to that of decentralized control; i.e. of determining the conditions under which one or more "local" output feedbacks can be applied to insure complete state controllability of the system through a selected input or set of inputs. The solution to the rather general question which is posed in [4] contains the elements for resolving the decentralized control question, as well as other "constrained" control

questions, in a new and efficient manner.

The particular treatment employed in [4] assumes knowledge of the (pxm) rational transfer matrix, T(s), which characterizes the dynamical behavior of the system under investigation. By then employing a relatively right prime factorization, $R(s)P(s)^{-1}$, of T(s) with P(s) in unique (upper right triangular) Hermite form, significant new insight is obtained relative to the changes which occur in all of the pxm elements of T(s) when any output y_i is fed back to any input, u_j through a constant gain element g_{ji}.

Algebraic Geometry

A portion of the research under the grant has focused on understanding and applying the methods of algebraic geometry in system theory in the context of the three key questions noted in the research objectives. A summary of various results and problems is given in [A] together with a fairly extensive bibliography.

The increasing application of the methods of algebraic geometry to systems problems (see, for example, the bibliographies in [A], [B]) has created, in the opinion of Falb, the need for a book which provides, in a motivated context, both the basic mathematical material and the relevant system theoretic results. While there is no lack of excellent mathematics texts on algebraic geometry, these books are not oriented to system theoretic applications and do not provide the applications motivation for studying the very difficult results of algebraic geometry. In essence, Falb believes that, just as optimal control in the early to mid-sixties was ready for a book combinding certain control and mathematical ideas (cf. [C]) so too is system theory ready for a book providing a combination of the relevant ideas of systems and algebraic geometry. A detailed outline (which follows) has been

developed and several chapters written. The book is a blending and extension of [A] and [D] and the basic approach is to relate the mathematical and systems ideas in such a fashion that the system theoretic motivation is apparent.

OUTLINE

I. Introduction

A. System

Internal, External Representations

B. Qualitative Properties

Key Questions

1) Orbits Under Equivalence

2) Spectral Structures Under Compensation

3) Essential Algebraic Elements

C. Routh-Hurwitz Theory

Rationality of a Proper Meromorphic Function

Stability of a Real Rational Function

D. Invariants

Basic Concept

II. Linear Systems

A. Notion of Linear System

B. External Description

Input-Output

Transfer Matrix

Algebraic Map

Mapping on a Riemann Surface

Pencils

C. Internal Description

State Representation

Polynomial Matrix Representation

Factorization

System Module

D. The Connecting Link

Space of Hankel Matrices

E. Equivalence

Unimodular Transformations

Linear Groups

F. Qualitative Properties

Controllability, Observability, Stability

III. Classical Groups of Symmetry and Moduli for Linear Systems

A. Theory of Invariants

Concept of Invariant

Ideas from Algebra

Polynomial Invariants

Some Algebraic Geometry

Hilbert-Mumford theory

B. Moduli for Linear Systems

Construction of the Moduli Space

Properties of the Moduli Space

Obstructions

Pole Assignment with Parameters

Of course, the book writing effort raises a number of research questions on which progress has been made. Several of these questions will be analyzed in the sequel.

The first question that has been studied extensively is the pole-assignment problem for systems with parameters. Let A,B be nxn and nxm matrices with entries in a field k and let $\psi_{A,B}$ be the map of k^{mn} into k^n given by

$$\psi_{A,B}(F) = (\mathrm{tr}(A + BF), \ldots, \mathrm{tr}(A + BF)^n) \tag{1}$$

This might be called the state-feedback trace assignment map. It is well-known that $\psi_{A,B}$ is surjective if and only if (A,B) is controllable. One method of approach is to use the following proposition:

PROPOSITION ([E]). The system (A,B) is controllable if and only if there is a K in k^{mn} and a b in Col (B) such that (A + BK,b) is controllable.

This reduces the problem to the scalar input case, i.e., to the case x = (A,b) and

$$\psi_x(f) = (\mathrm{tr}(A + bf)^i) = \left(\sum_{j=0}^{i} \binom{\ }{\ } \mathrm{tr}(A^{i-j} (bf)^j)\right) \tag{2}$$

with i = 1, ..., n. If $\phi_x(f)$ is given by

$$\phi_x(f) = (\mathrm{tr}(A^{i-1}bf)) \tag{3}$$

with i = 1, ..., n, then it is easy to see that ψ_x is surjective if and only if ϕ_x if is surjective. But ϕ_x is a linear map of k^n into k^n and hence is surjective if and only if $\partial\phi_x / \partial f$ is nonsingular, i.e., if and only if det $[bAB\ldots A^{n-1}b[\neq 0$. Now the problem considered is the following: let x = x(p) = (A(p), B(p)) be a family of linear systems depending algebraically on a parameter p and consider the trace assignment map

$$\psi_x(F,p) = (\mathrm{tr}(A(p) + B(p)F)^i) \tag{4}$$

with $i = 1, \ldots, n$. In this case, Ψ_x is a map of $k^{mn} \times P$ into k^n and the issue is to determine the portion of the range of ψ_x which is independent of P (or, in other words, how many poles can be assigned independently of the parameter p?). The work extends and generalizes that of Eldem ([F]). The approach used is the following: first, a lemma is established showing that (generically, at least) the dependence on p may be assumed linear; then a result analogous to the proposition (Heymann's lemma) is used to reduce to the scalar input case; and, finally, the scalar input case is analyzed by a process similar to that used in going from the map ψ_x of equation (2) to the map ψ_x of equation (3). The key point is to determine the intersection of the range of $\psi_x(F,p)$ and the variety $D_p \psi_x(F,p) = 0$. The dimension has been calculated using intersection theory.

State Feedback

Results concerning the action of state feedback were developed in [G]. If $T(s) = R(s)P^{-1}(s)$ is a proper transfer matrix with R(s), P(s) right coprime, then (R(s), P(s)) may be viewed as "homogeneous coordinates" of a point under right multiplication via the unimodular group U_m. The orbits under state feedback are represented by the action of stabilizer subgroups U_∂ where $\partial = \partial_1, \ldots, \partial_m \}$ are the Kronecker indices. The subgroup U_∂ is the semi-direct product of a normal subgroup U_N (the unipotent radical) and a reductive subgroup U_G acting on U_N via inner automorphisms. A special representation of U_∂ played a key role in the development. Now, the problem studied is the following: suppose that $(R_1(s), P_1(s))$ is a subsystem of (R(s), P(s)) and consider the subgroup U_∂^1 of U_∂ leaving (R_1, P_1) invariant; then (i) what is the structure of U_∂^1? (ii) what is the relation of U_∂^1 to U_{∂}^{1} (the stabilizer subgroup corresponding to (R_1, P_1))? (iii) if (R_1P) splits into subsystems (R_i, P_i), does U_∂ split into the U_∂^i in an appro-

priate manner? Preliminary results have been obtained by analyzing the representation of U_∂ and its subrepresentations and by analyzing the representation of U_∂ induced by that of U_∂^1. This work could lead to an understanding of the action of feedback on interconnected systems.

Publications and Presentations

1 H. Elliott and W. A. Wolovich, "A Parameter Adaptive Control Structure for Linear Multivariable Systems" Proceedings 1980 JACC San Francisco, CA, August, 1980. Also submitted to the IEEE Transactions in Automatic Control.

2 W. A. Wolovich, "Multi-Purpose Controllers for Multivariable Systems" Proceedings 1980 JACC San Francisco, CA, August, 1980. Also to appear in the Special Issue on Linear Multivariable Control Systems of the IEEE Transactions on Automatic Control, February, 1981.

3 H. Elliott and W. A. Wolovich, "A Frequency Domain Model Reduction Proceedure" Automatica, March 1980 pp. 167-178.

4 W. A. Wolovich, "Scalar Output Feedback in Linear Multivariable Systems" Brown University Division of Engineering Technical Report #ENG SE 80-1, September, 1980.

5 W. A. Wolovich, "Complex Compensation of Multivariable Systems" presention at Colorado State University, Fort Collins, Colorado, August 12, 1980.

6 P. L. Falb, "Applications of algebraic geometry in system theory", lectures at MIT lab. for Information and Decision Sciences, 1980.

7 P. L. Falb, "Applications of Algebraic Geometry in System Theory", in preparation.

8 P. L Falb, Pole-assignment with parameters, to be submitted to SIAM J. on Control.

Personnel

1. William A. Wolovich (co-principal investigator)
2. Peter L. Falb (co-principal investigator)
3. Tzyy-Jer Huang (graduate student)

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